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Research on the Teaching Model of Advanced Mathematics Based on Problem-Based Learning (PBL)

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Abstract: This study examines the application of Problem-Based Learning (PBL) in Advanced Mathematics education to enhance student engagement and conceptual understanding. By integrating real-world problems and collaborative inquiry, the PBL model shifts the focus from passive learning to active knowledge construction. The research highlights the model's effectiveness in fostering critical thinking, problem-solving skills, and interdisciplinary connections. Findings suggest that PBL not only improves learning outcomes but also transforms students into proactive learners. The study proposes future directions, including the development of PBL case databases and blended learning approaches, to further optimize mathematics instruction.

Keywords: Advanced mathematics; Problem-based learning; Education

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1. Introduction

Advanced Mathematics is a core course for STEM majors, yet traditional teaching methods often lead to low student engagement, disconnection from real-world applications, and over-reliance on exam-based assessments. Problem-Based Learning (PBL), originally developed in medical education, offers a promising solution by using authentic problems to drive collaborative inquiry and knowledge construction [1]. This study investigates PBL's adaptation to Advanced Mathematics, addressing key challenges: (1) aligning abstract mathematical concepts with real-world contexts, and (2) designing scaffolded problems for diverse learners. The research aims to validate PBL's effectiveness in enhancing conceptual understanding, problem-solving skills, and interdisciplinary application. By integrating constructivist and pragmatic theories, the proposed model seeks to transform passive learning into active exploration, offering practical strategies for educators and improving learning outcomes for students.

2. Theoretical foundations of the PBL model

2.1. Learning theory foundations

2.1.1. Constructivism

Core Tenets:

Piaget emphasized that knowledge is actively constructed through individual-environment interactions.

Vygotsky highlighted the role of social interaction in cognitive development [2].

Implications for Mathematics:

The abstract nature of mathematical knowledge requires authentic problem contexts (e.g., “applying graph theory to logistics route optimization”) to facilitate meaningful construction.

2.1.2. Pragmatic education theory

Core Tenets:

“Education as the reconstruction of experience”, with a focus on “learning by doing” [3].

PBL Application:

Embeds mathematical tools (e.g., differential equations) into real-world problems (e.g., epidemic prediction), avoiding decontextualized symbolic instruction.

2.2. Instructional implementation theories

2.2.1. Situated cognition

Core Tenets:

Learning occurs through participation in communities of practice [4].

PBL Design Example:

Role-playing tasks (e.g., “engineer-mathematician collaboration” for bridge load calculations) reinforce awareness of mathematics’ professional utility.

2.2.2. Cognitive load theory (Sweller)

Core Tenets:

Problem decomposition reduces intrinsic cognitive load; scaffolding (e.g., providing initial conditions for differential equation modeling) supports learning [5].

3. PBL teaching model design for advanced mathematics

3.1. Design philosophy and principles

Based on constructivist theory and the characteristics of mathematics, the PBL model follows a design philosophy of “One Core, Dual Integration, Three-Dimensional Objectives.”

One Core:

Driven by authentic problems, the model completes a closed-loop process of “problem scenario → mathematical modeling → solution verification → extended application” to achieve knowledge construction.

Dual Integration:

The PBL pedagogy is deeply integrated with the disciplinary features of advanced mathematics (abstraction, logical rigor, and applicability).

Three-Dimensional Objectives:

Knowledge acquisition (conceptual understanding/computational skills), cognitive development (abstraction/reasoning/modeling), competency cultivation (innovation/collaboration).

Design principles include:

1. Authentic Problem Principle:

Select real-world cases from engineering, economics, and physics (e.g., “COVID-19 transmission prediction” for differential equations).

2. Cognitive Scaffolding Principle:

Problem design follows the mathematical cognitive progression: intuitive perception → formal definition → symbolic computation → practical application.

3. Scaffolding Support Principle:

Provide visual aids for abstract concepts (e.g., using GeoGebra to illustrate the geometric meaning).

Formative Assessment Principle:

Establish a multi-dimensional evaluation system incorporating process documentation, peer evaluation, and defense performance.

3.2. Overall framework design

The model adopts a “Dual-Cycle Four-Phase” operational structure (as shown in **Figure 1**):

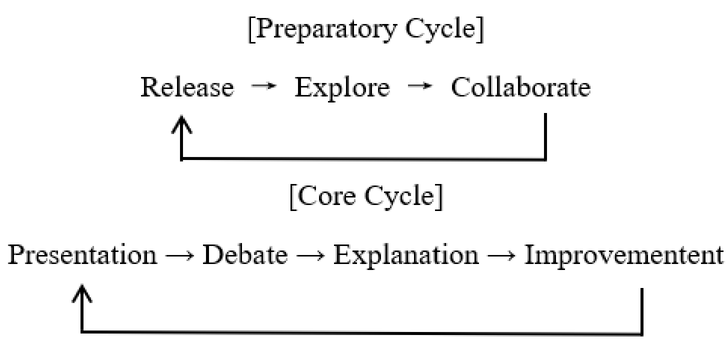

Figure 1. “Dual-cycle four-phase” operational structure

Typical teaching unit time allocation (in units of 4 class hours):

Introduction of problem scenarios (0.5 class hours) - Extracurricular independent exploration (6–8 hours) - Classroom collaborative discussion (2 class hours) - Achievement display and evaluation (1 class hour) - Reflection and expansion extension (0.5 class hours)

3.3. Key element design

3.3.1. Problem design matrix

Establish a two-dimensional design framework of “knowledge dimension difficulty level.” Typical Case Example (Using Surface Integrals):

Problem Context: The reentry capsule of a spacecraft requires a special thermal protection coating, where the coating thickness is correlated with surface curvature. Given the parametric equation of the reentry capsule’s surface: $z=x^2+y^2(-1\leq x,y\leq 1)$.

Requirements:

(1) Calculate the Gaussian curvature of each point on the surface;

(2) Assuming that the coating cost per unit area is C=2+|K| (K is curvature), find the minimum total cost;

(3) The process treatment scheme of curvature mutation is discussed.

This problem integrates multiple knowledge domains, including curvature computation, surface integration, and optimization theory, while requiring MATLAB programming implementation and engineering decision-making.

3.3.2. Learning support system

Construct a trinity support system:

1. Digital Resource Kit

Core Resources: Micro-lecture videos (deconstructing key/difficult concepts);

Extended Resources: 3Blue1Brown animations, MIT OpenCourseWare links;

Tool Resources: MATLAB code templates, Symbolab calculation guides.

2. Interactive Learning Platform

Online forum with sections, Jupyter Notebook-based interactive math labs.

3. Expert Consultation Network

Cross-disciplinary mentor pool (physics, economics, etc.), "Friday Q&A Sessions" for collective problem-solving.

3.3.3. Teacher facilitation strategies

Adopting a "Gradual Scaffolding" approach:

1. Initial Phase: Metacognitive Prompting

Guiding questions:

"Which previously learned concepts does this problem connect to?" "What information sources will you explore?"

2. Intermediate Phase: Cognitive Intervention

For struggling groups:

• Provide hints instead of solutions (e.g., "Consider revisiting Green's Theorem")

• Guide error correction through Socratic questioning:

"Does this solution maintain physical unit consistency?" "What would happen if the boundary conditions changed?"

3. Final Phase: Deep Reflection

Structured protocols:

• Reflection journals with prompts:

"What was the most counterintuitive discovery?" "How would you approach this differently next time?"

3.4. Implementation process design

3.4.1. Preparation phase

1. Diagnostic Assessment: conduct pre-test questionnaires to evaluate students' prior knowledge foundations;

2. Group Formation Strategy: adopt "Heterogeneous Homogeneous Grouping" with each team comprising: 1 student with strong mathematical foundations; 2 students with intermediate proficiency; 1 student with outstanding practical skills;

3. Learning Contract Establishment: formalize team learning agreements specifying mandatory weekly discussion sessions (minimum 1 session/week).

3.4.2. Implementation phase

A 6-Step Cyclic Process Demonstrated via "Gradient Descent Optimization Problem":

1. Problem Representation: present optimization task:

$\min f(x,y)=x^2+y^2+2\sin(x+y)$.

Guiding Students to Translate Engineering Optimization Problems into Mathematical Formulations

2. Solution Design: Each group shall submit the research plan, including the proposed algorithm (steepest descent/Newton method), convergence analysis ideas, and programming implementation path.

3. Exploratory Implementation: Inquiry implementation: the teacher demonstrates the basic version of MATLAB in class, and the group improves the algorithm after class (such as adding adaptive step size)

4. Midterm Defense: Each group will report the current progress, focusing on the mathematical difficulties encountered (such as the irreversibility of the Hessian matrix) and the tried solutions;

5. Solution Refinement: directional adjustments based on feedback, supplemental learning (convex optimization theory)

6. Final Demonstration: submit a complete technical report, and conduct the algorithm performance comparison experiment.

3.4.3. Evaluation phase

The "five-dimensional radar chart" assessment system is shown in **Figure 2**.

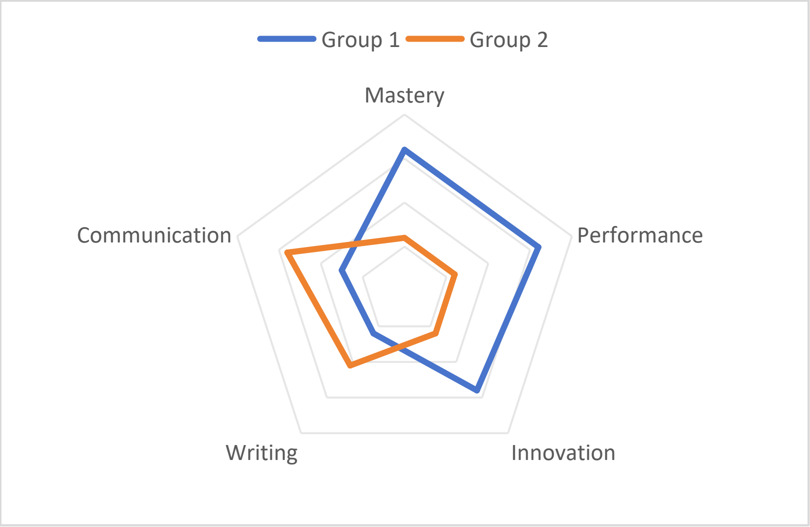

Figure 2. The "five-dimensional radar chart" assessment system: Mastery (Test results), Performance (Completeness of learning log), Innovation (Uniqueness of solution), Writing (Team members' mutual evaluation), Communication (Q&A quality)

4. Empirical research

4.1. Participants

This study employed a quasi-experimental design with two parallel classes enrolled in "Advanced Mathematics" during the first semester of 2024–2025.

Experimental group (PBL, n=48): Implemented the PBL teaching model;

Control group (traditional, n=45): Maintained lecture-based instruction.

Baseline equivalence (pre-test):

Gender ratio: PBL: 25 male/23 female, Control: 24 male/21 female;

College entrance math scores: PBL: M=125.3, SD=8.7, Control: M=123.9, SD=9.2

t=0.782, P=0.436 (no significant difference).

4.2. Instruments

Two data collection tools were used: Achievement tests (pre/post):

1. Academic achievement test: prepare pre-test and post-test papers, covering core knowledge points such as limit, derivative, and integral, with Cronbach's α coefficient of 0.82 and 0.85, respectively.

2. PBL teaching effect questionnaire: it contains three dimensions: learning interest (5 questions), autonomous learning ability (6 questions), and team cooperation (4 questions). It is scored on a Likert 5-point scale, and the total table α=0.89.

4.3. Implementation procedure

The 16-week intervention followed this protocol:

1. Pre-test Phase (Week 1): At the beginning of the term, the two groups of students were tested for the level of advanced mathematics.

2. Instructional Delivery (Weeks 2–15):

PBL: Conducted biweekly PBL seminars featuring six authentic problem scenarios (e.g., "Modeling and Analysis of COVID-19 Transmission"). The instructional process followed: Problem presentation → Group discussion → Independent inquiry → Solution demonstration → Reflective evaluation.

Control: Received traditional lecture-based instruction supplemented with problem-solving sessions.

3. Data Collection (Week 16): Researchers administered a post-test at the end of the semester and distributed questionnaires (yielding 46 valid responses from the PBL group and 43 from the Control group).

4.4. Data analysis results

4.4.1. Result comparison

As shown in **Table 1**, the ANCOVA (controlling for pretest scores) indicated that the posttest scores of the experimental group were significantly higher than those of the control group ($F=9.327$, $p=0.003$, $\eta^2=0.112$). The experimental group exhibited an even more pronounced advantage in word problem-solving scores (t=3.892, P<0.001).

Table 1. Comparison of student performance between groups (Mean ± SD)

Group	Pre-test scores	Post-test scores	Increased scores
PBL	72.5±9.1	83.7±7.8	11.2±5.9
Control	71.8±8.7	78.3±8.4	6.5±6.3

4.4.2. Questionnaire results

Analysis of the questionnaire data (**Table 2**) revealed that students in the experimental group showed significant improvement across all dimensions ($P < 0.01$), with effect sizes (Cohen's d) ranging from 0.67 to 0.92, indicating moderate to large effects. The greatest improvement was observed in the "Mathematics Application Confidence" dimension (mean difference = 1.12).

Table 2. Pre-post comparison of PBL teaching effectiveness across questionnaire dimensions

Dimension (s)	Pre-test mean	Post-test mean	Mean difference	t	Cohen's d
Interest	3.15±0.72	4.02±0.68	0.87	6.892	0.83
Self-Regulated Learning	3.28±0.81	4.05±0.73	0.77	5.673	0.71
Teamwork	3.41±0.69	4.23±0.65	0.82	7.112	0.92

5. Conclusion

This study confirms the efficacy of Problem-Based Learning (PBL) in advanced mathematics education. The PBL model successfully addresses key teaching challenges by engaging students with authentic, interdisciplinary problems and collaborative learning processes. Results demonstrate significant improvements in students' conceptual understanding, problem-solving skills, and ability to apply mathematical knowledge in practical contexts. The framework's emphasis on scaffolded problem design and process-oriented assessment offers a viable alternative to traditional lecture-based instruction. These findings establish PBL as a valuable pedagogical approach for transforming advanced mathematics into a more dynamic and application-focused discipline.

Disclosure statement

The author declares no conflict of interest.

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