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Research on Dimension Analysis of Spectative Buildings Based on Fractal Theory

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Abstract: Based on the fractal theory, this paper takes the form of performing architecture as the research object, and systematically discusses the application value of fractal dimension in architectural design. By expounding the self-affine, self-similarity, and iterative generation characteristics of fractal geometry, the Box-Counting Dimension method is introduced as a quantitative tool to measure the dimensions of the roof plane, facade, and spatial shape of Wuzhen Grand Theatre and Harbin Grand Theatre. The research shows that the geometric complexity of Wuzhen Grand Theater in the "fifth façade" and multi-faceted façade is significantly higher than that of Harbin Grand Theater, and its morphological design is more inclined to echo the texture of the surrounding water towns. The Harbin Grand Theater realizes the dialogue with the natural environment with simple nonlinear lines. The research proves that fractal dimension can effectively quantify the complexity of architectural form, provide a scientific basis for the form design, environmental integration, and form interpretation of performance architecture, and expand the mathematical analysis dimension of architectural form design.

Keywords: Fractal dimension; Fractal geometry; Performing architecture; Architectural form; Architectural design

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1. Introduction

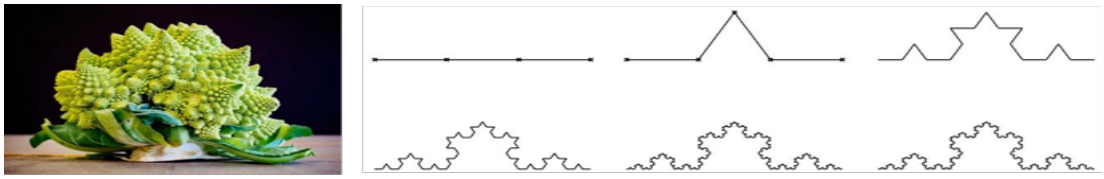
Since the fractal theory was proposed by Mandelbrot in the 1970s, it has gradually become an important mathematical tool for describing the complex shapes and structures in nature and human creation [1]. As an important part of nonlinear science, fractal theory provides a new perspective and method for architectural design with its unique self-similarity and scale hierarchy [2]. In the field of architecture, the application of fractal theory has gradually expanded from the early theoretical discussion to the actual architectural design and analysis, especially in the aspects of architectural form analysis, spatial layout optimization, and urban and rural settlement system research [3]. Performance architecture is a kind of public building with both performance space and viewing space. It is not only the medium of social culture and art communication, but also the carrier of urban cultural activities [4]. Its form design needs to take into account both artistic expression and spatial efficiency. Traditional analysis methods of performing architecture mostly rely on qualitative description and experience summary, and there are obvious deficiencies in quantifying spatial complexity and audience experience [5]. Therefore, the

dimension analysis of the performance building based on the fractal theory not only helps to reveal its inherent spatial law, but also provides a new theoretical perspective for the optimization of building performance.

2. Fractal geometry and dimension

The concept of "fractal" was first proposed by Benoit B. Mandelbrot in 1975. It originated from the understanding of irregular and fragmented graphics in nature. The word fractal itself has two meanings: "broken" and "irregular" [6]. In 1904, the Swedish mathematician Koch first constructed the geometric concept of the "Koch curve". Next, the Koch curve of the third iteration is constructed as a case to explain its construction process. The process can be divided into three core stages: first, the basic graph is a single straight line segment; then, the outward convex folding operation is performed at the third equidistant point of the original line segment; finally, the same proportion of outward extension folding is continuously implemented for all new line segments. Through infinite iterations of the operation process, the Koch curve shape with complex fractal structure is finally formed.

Fractal geometry presents three typical characteristics: self-affine, self-similarity, and iterative generative (as shown in **Figure 1**). In the field of architecture, the application of the fractal concept, based on comprehensive function, streamline, and scale, focuses on exploring and refining the corresponding design methods from the composition idea and organizational logic of fractal graphics, which provides an effective scheme for innovative architectural form design [7].



(a) Self-affine of Roman cauliflower

(b) The self-similarity and iterative generation of Koch curve

Figure 1. Self-affine, self-similarity and iterative generation of fractal geometry

Figure 1. Self-affine of Roman cauliflower along with the self-similarity and iterative generation of Koch curve

The fractal dimension is a measure of the spatial structure and morphological complexity of spatial objects at a certain scale. The larger the value, the higher the complexity of the object [8]. It can relatively accurately reflect the effectiveness of the space occupied by complex shapes, measure the degree of irregularity of complex shapes, and realize the expansion of the European integer dimension. Its value is positively correlated with the spatial morphological abundance of the shape. Among them, the box-counting dimension method (Db, Box-Counting Dimension) is widely used in the field of architectural form analysis due to its advantages of simple calculation and convenient operation. Through the two-dimensional square grid of different scales, the method can effectively establish the geometric correlation with the architectural plane, elevation, and detail texture (corresponding to the architectural form of the three-dimensional cube box array) and other morphological information, and then analyze its formal connotation. Its core principle is to cover the target geometry with a square grid with a decreasing size, and to count the number of non-empty grids occupied by the graphics at each scale. As the mesh size decreases step by step, the number sequence of non-empty meshes at different scales is obtained, and the box dimension value of the measured object is calculated following **Formula 1**.

$$D_b = \frac{\log N_{n+1} - \log N_n}{\log\left(\frac{1}{r_{n+1}}\right) - \log\left(\frac{1}{r_n}\right)} \tag{1}$$

Of which, D_b——value of box dimension; r——side length of box; n——1,2,3,4……; N——number of boxes

The derivation of the formula is based on the linear regression relationship in the double logarithmic coordinate system. If the goodness of fit is good, the object is considered to have self-similarity, which satisfies the research conditions of fractal theory [9, 10].

3. Dimensional analysis

As one of the core branches of fractal theory, fractal dimension quantitatively characterizes the filling ability of objects to space and the irregular characteristics of their shapes through non-integer dimensions. In the field of architecture, the box-counting method has become an important tool for quantifying the geometric shape of buildings. With the help of the calculation program constructed by the Rhino-Grasshopper platform, the box-counting method can be used to calculate the plane, elevation, and overall shape of the performing buildings, and then reveal the mathematical rules of its morphological design, and provide a quantitative basis for design decision-making. Taking Wuzhen Grand Theatre and Harbin Grand Theatre as the research objects, by measuring the box dimension values of their roof plane, façade, and shape, the differences in the complexity of architectural forms between the two can be quantitatively compared, and the interaction between architecture and environment and morphological treatment strategies can be discussed in depth. Among them, the roof plane, as the "fifth elevation" of the building, can directly present the interaction between the overall layout of the building and the surrounding environment; facade structure and shape modeling are the intuitive expression carriers of architectural form, and the difference of box dimension value can reflect the control ability of design methods to spatial complexity. Therefore, the dimension analysis strategy of this paper is mainly aimed at the roof plan, façade, and spatial form of the performing building. It can be used not only for the quantitative interpretation of the complexity of the form of the design work, but also for the adjustment and control of each part of the architectural form in the process of project design.

3.1. Plane dimension analysis

In the analysis of plane dimension, the box dimension method is used to quantitatively evaluate the geometric complexity of the "fifth façade" of the performance building, which can reveal the architect's consideration in the organizational design of the overall plane outline and the built texture around the site, and explore the degree of echo and integration of architectural form and site environment through data means. Taking Wuzhen Grand Theatre and Harbin Grand Theatre as examples (**Figure 2**), the calculation data show that the box dimension value of the roof plane of Wuzhen Grand Theatre is 1.749102, and that of Harbin Grand Theatre is 1.617411, indicating that the geometric complexity of the "fifth façade" of the former is about 8.14 % higher than that of the latter (**Table 1**).

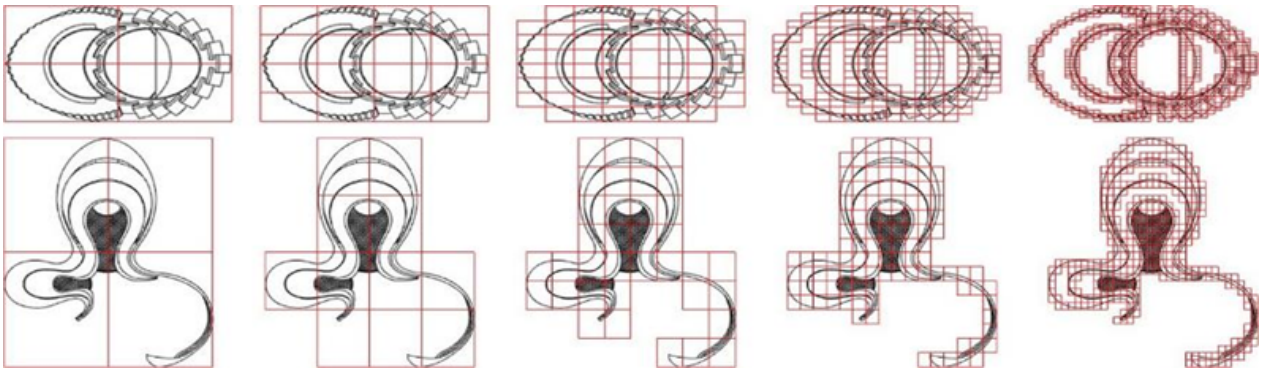
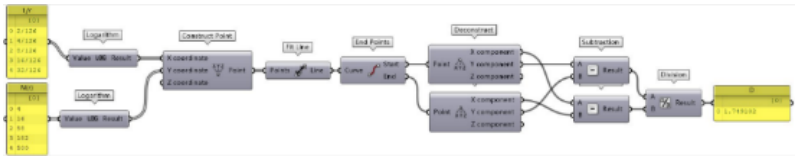
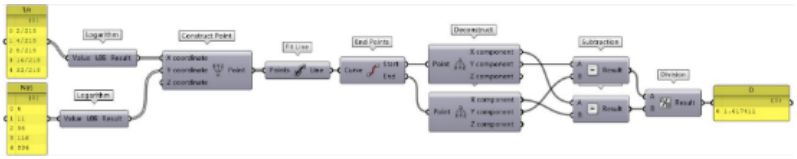


Figure 2. Box-dimensional numerical calculation process of roof plane of Wuzhen Grand Theatre and Harbin Grand Theatre

Table 1. Roof plane box dimension value of performing building case

Case of architectural	Value record		Fitting evaluation
Wuzhen Grand Theatre (L_1=126)	r1=L1/2	N1=4	 Dr=1.749102
	r2=L1/4	N2=16	
	r3=L1/8	N3=58	
	r4=L1/16	N4=182	
	r5=L1/32	N5=500	
Harbin Grand Theatre(L2=218)	r1=L2/2	N1=4	 Dr=1.617411
	r2=L2/4	N2=11	
	r3=L2/8	N3=36	
	r4=L2/16	N4=116	
	r5=L2/32	N5=334	

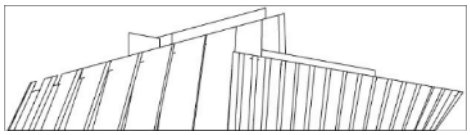
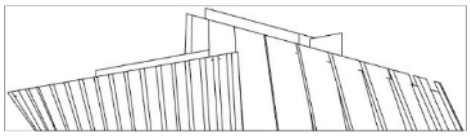
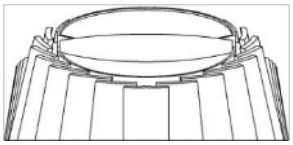
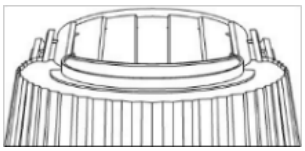
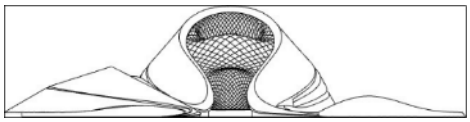
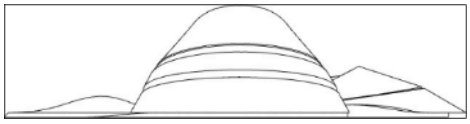
On the basis of the comparison of the box dimension values of the two roof planes, the design logic can be further explained by combining the surrounding environmental characteristics of the two case sites: Wuzhen Grand Theater is located inside the Xizha Scenic Area, and its site is close to the typical Jiangnan water village environment. In the close range, it is dominated by small-scale traditional dwellings (the slope roof shape is matched with the architectural language of small green tiles). The building complex presents a densely scattered layout feature, forming an environmental texture with high geometric complexity. The higher box dimension value of the roof plane of the Grand Theatre directly reflects the response of the design to the existing environmental information of the site, and strengthens the correlation with the surrounding traditional texture through the morphological complexity. The Harbin Grand Theater is located in the island area on the banks of the Songhua River. It is located in the open waterfront natural landscape in the near-scale range. Its design adopts a simple nonlinear roof geometry, and realizes direct dialogue with the remote natural environment with a low-complexity plane composition. Through the differentiated geometric complexity strategy, the two have completed the fit with their respective site environment in the top-level interface dimension. The Wuzhen project echoes the dense humanistic texture with high dimensions, and the Harbin project integrates the natural landscape with low dimensions, and finally achieves the organic unity of architectural form and site context.

3.2 Facade dimension analysis

As an intuitive presentation of the overall external form of the building, the building facade occupies an important position in the architectural form design. Its design can not only emphasize the unified coordination of interface forms, but also form a comparative effect according to the actual situation. With the help of fractal dimension to quantify the geometric form of the facade of the performing building, the construction method of its external form interface can be revealed and interpreted, and then the design strategy of the facade of the performing building based on fractal dimension can be discussed. Taking Wuzhen Grand Theatre and Harbin Grand Theatre as examples, the calculation data shows (as shown in **Table 2**) the average box dimension value of each facade of Wuzhen Grand Theatre is 1.779031, and the geometric form information of the building on different azimuth interfaces is generally close, and the main interface box dimension value for water surface and scenic tourists is the highest. The box dimension value of the facade at the main entrance of Harbin Grand Theater for tourists is more than 1.65, while the back facade of the building is simple and the box dimension value is the lowest, which is 6.83% different from the overall average. The box dimension values of each facade are close as a whole, reflecting the processing and

organization methods of flexible expression according to functions on the basis of harmony and unity in the form of architectural interface.

Table 2. The box dimension value of the facade of the performing building case

Case of architectural	The box dimension value of each facade		The average box dimension value of the outer facade
Wuzhen Grand Theatre		D_{e1}=1.770262	D_e=1.779031
		D_{e2}=1.770262	
Wuzhen Grand Theatre		D_{e3}=1.717857	D_e=1.779031
		D_{e4}=1.857744	
Harbin Grand Theatre		D_{e1}=1.658483	D_e=1.613071
		D_{e2}=1.635581	
		D_{e3}=1.655377	
		D_{e4}=1.502842	

As far as the average box dimension value of the facade of Harbin Grand Theater is 1.613071, compared with the measured value of Wuzhen Grand Theater, the curve outline of its facade drawn by free nonlinear lines and the unified material treatment of the overall large-area continuous interface shape a more concise architectural image, thus obtaining a lower overall numerical performance. On the other hand, the two building cases have obtained relatively accurate data results in the calculation of the complexity of the facade geometry in different directions, which further verifies that the facade dimension analysis method of the form design of the performing architecture has good practicality and feasibility in quantifying, comparing and analyzing the architectural form, and can provide effective help for the form design, interpretation and analysis of the performing architecture to a certain extent.

4. Conclusion

Based on the fractal theory, this paper systematically analyzes the dimension of the performing architecture, quantifies the complexity of the plane, facade, and spatial form of the building roof by the box dimension method, and reveals the mathematical rules of the morphological design of the performing architecture. The box-counting dimension method can effectively measure the geometric complexity of architectural plane and elevation, and provide data support for morphological design. The difference in morphological complexity between Wuzhen Grand Theatre and Harbin Grand Theatre reflects the designer's differentiated response strategy to the site environment. As a quantitative tool, fractal dimension can objectively evaluate the degree of integration of architectural form and environment, and provide accurate control parameters for design optimization.

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Disclosure statement

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