Scientific and Social Research, 2025, Volume 7, Issue 8
https://ojs.bbwpublisher.com/index.php/SSR
Online ISSN: 2981-9946
Print ISSN: 2661-4332

Enhancing Submersible Safety: Predictive Modeling and Search Model for Lost Submersibles

Yihang Wang*, Hongwei Chen, Xinmu Xu

Portland Institute, Nanjing University of Posts and Telecommunications, Nanjing 210023, Jiangsu, China

**Author to whom correspondence should be addressed.*

Abstract: The Titan submersible accident has raised global concerns regarding submersible safety. To mitigate such risks and enhance survival probabilities, this study develops a safety support system aimed at predicting submersible trajectories and optimizing search operations. A Submersible Position Prediction Model is established, incorporating seawater salinity, temperature, current, and seabed elevation data from the Ionian Sea. Force analysis based on computer simulations is used to generate minute-by-minute positional data, resulting in a probability distribution of its location. Model validation is conducted using Caribbean Sea data. Furthermore, a Two-Dimensional Kernel Density Estimation Search Model is proposed to minimize search time for a lost submersible. The method partitions the probability distribution into four regions, each searched by a robot moving outward from the center, significantly improving search efficiency. A function relating search success probability to time is derived based on robot speed and detection range. The results show that the lost submarine can be found within 1.3 hours. Finally, a multi-submersible position prediction model is introduced, which updates the dive path of one submersible based on real-time positions of others, enhancing coordination and emergency response capabilities.

Keywords: Computer simulation; 2D KDE; Submersible position prediction; Search model; Force analysis

Online publication: September 17, 2025

1. Background

On June 18, 2023, a U.S. tourist submarine lost contact while exploring the wreck of the Titanic, which shocked the world. After the loss of contact, no one knew the location of the submersible. The search lasted for a long 4 days, and ultimately, only the wreckage of the submersible was found. This tragedy was caused by inadequate safety measures of the submersible. Concerns have been raised about the safety of the submersible.

In order to sustain the development of the industry, more stringent safety measures are necessary. The authors build a predictive model for the position of submersibles to prevent loss of contact and to speed up the search. After careful analysis of this issue, the authors have identified six main sub problems that need to be addressed in this article: (1) Establish a model to predict the change of submersible position over time. (2) Establish a model to search the submersible and determine the probability of finding a submersible as a function of time. (3) Establish the mutual warning model of multiple submersibles.

2. Methodology and core concepts

According to existing research, there are two main methods for predicting the trajectory of a submersible: dynamic simulation and neural network prediction [1]. This article adopts a method similar to dynamic simulation. During the simulation of the submersible dive process, data on salinity, temperature, ocean currents, and seabed elevation were first collected. Based on these data, the density and resistance of seawater were calculated, enabling the determination of the submersible's state at any given moment. Through multiple simulation runs, the distribution of potential diving paths was derived. This model also explains the types of data required for predicting the submersible's position accurately.

For the design of search methods for a lost submersible, a two-dimensional kernel density estimation (2D KDE) was used to obtain the probability density distribution of its location. After simplifying the probability density distribution, a search model for searching elliptical regions was established. The elliptical area was divided into four sectors, and each was assigned a robot to conduct an inward-to-outward search. Based on the robot's speed and detection radius, a function relating search success probability to time was established.

There is existing work on submersible search models, but there is a lack of safety measures for coordinating multiple submersibles [2]. Regarding the influence of multiple submersibles on the diving process, a mutual positioning method among submersibles was proposed. Using the coordinates determined through inter-submersible communication, Model I was updated to reduce errors in trajectory prediction.

3. Model I: Submersible position prediction model

3.1. Force analysis of submersible

The state of the seawater is reflected in its density. According to the equation of state of seawater summarized, the density of seawater ρ can be derived [3]. The parameters of the function are seawater salinity $S(g/kg)$, temperature $T(℃)$ and pressure $P(Pa)$. Density will increase with increasing salinity, decreasing temperature, and increasing pressure; otherwise, it will decrease, i.e. $\rho \propto S\, T^{-1}, P$.

Obtain the density of seawater $\rho_{STP}(kg/m^3)$ at pressure P:

$$\rho_{STP} = \rho(S,T,P) = \frac{\rho(S,T,0)}{1-\frac{P}{K(S,T,P)}}(kg/m^3) \tag{3.1}$$

For a given seawater depth, the pressure is approximated as that induced by surface seawater density at an equivalent depth. After obtaining the density of seawater, the thrust ocean currents and the resistance of seawater on the submersible can be solved.

The formula of seawater resistance is:

$$f = \frac{1}{2}C\rho_{STP}S_s v^2(N) \tag{3.2}$$

Where C is the resistance coefficient, ρ_{STP} is seawater conditions, it can be monitored on-site; $v(m/s)$ is the area of the submersible; $S_s(m^2)$ is the fluid velocity [4].

When submersibles dive without power, they are pushed by ocean currents. The impact force F_C exerted by water on an object is related to the water velocity $v_C(m/s)$ and cross-sectional area S_s. The formula is:

$$F_c = \frac{1}{2}\rho_{STP}S_s v_c{}^2(N) \tag{3.3}$$

Next, conduct a force analysis on the submersible. In the horizontal direction, the submersible is affected by its own thrust F_s, ocean current thrust F_C, and seawater resistance $f(N)$. The magnitude of the submarine thrust plus seawater resistance is equal to the ocean current thrust, and the forward direction of the submarine affected by the ocean current is equal to the direction of the ocean current. The formula is:

$$F_c = F_s + f(N) \tag{3.4}$$

In the vertical direction, the submersible's own gravity G is equal to the buoyancy F_b plus the water resistance f:

$$G = F_b + f(N) \tag{3.5}$$

Where the buoyancy of the submersible is proportional to its own volume $V(m^3)$ and seawater density ρ_{STP}. Its formula is as follows:

$$F_b = \rho_{STP}gV(N) \tag{3.6}$$

Where g is the gravitational acceleration of the region, which is .

3.2. Submersible diving simulation

In order to get the submersible position prediction model, a computer simulation method is used to calculate the speed and direction of the submersible's movement at each minute of diving, based on the surrounding conditions and its own situation.

Divide the diving process of the submersible into several discrete motion processes based on time, with an interval of one minute between each motion process. From the beginning of diving, the time t for the submersible to detach from the host ship is 0, the submersible position is $p(0,0,0)$.

According to formulas A and B, the magnitude and direction of the horizontal and vertical motion velocities of the submersible at t minute can be calculated and synthesized into a motion vector $v(x,y,z)$, where x,y,z are coordinates of the submersible at time t, i.e. $x(t),y(t),z(t)$.

The displacement of the submersible within t minute is $D(x,y,z)$, where $D(x,y,z)=v(x,y,z)\times 60$. Take the partial derivative of and obtain the velocity in the axis direction:

$$v_x(t) = \frac{\partial \boldsymbol{D}(x,y,z,t)}{\partial x} \tag{3.7}$$
$$v_y(t) = \frac{\partial \boldsymbol{D}(x,y,z,t)}{\partial y}$$
$$v_z(t) = \frac{\partial \boldsymbol{D}(x,y,z,t)}{\partial z}$$

Force analysis is performed on the submersible using its current position, denoted as $p(x,y,z,t)$, enabling estimation of its coordinates at the next one minute $p(x,y,z,t+1)$. The formula is:

$$x(t+1) = x(t) + v_x(t) \times 60$$
$$y(t+1) = y(t) + v_y(t) \times 60 \quad (3.8)$$
$$z(t+1) = z(t) + v_z(t) \times 60$$

According to the publicly available online seabed elevation map, the depth $d(x,y)$ of the diving area at coordinates (x,y) can be obtained. At a certain time t, if $z(t=1) < d(x,y)$, then the submersible has already made contact with the seabed, and the simulation stops.

It is worth noting that due to the operation of the submersible's own engine and the small changes in ocean currents, random position offsets are also needed. The effect of the offset at time t will be reflected in $p(x,y,z,t+1)$. The algorithm is shown in **Table 1**.

Table 1. Algorithm: The process of submersible diving

Input: $S, T, P, d(x,y), vc, S_s, V$
Output: submarine position Set the starting point to $p(0,0,0)$.
for t=1 to ∞ do
Calculate the seawater density ρ_{STP} at position (x,y,z) based on (3.1).
Calculate the diving speed and direction based on the situation of the submersible, based on (3.4), (3.4). Add random variations to the obtained speed $v(x,y,z)$ to simulate ocean current fluctuations and engine operation.
Calculate the position in based on (3.8).
If the t+1, end loop.
$z(t+1)$ end $d(x,y)$

3.3. Result of Model I

The diving process is simulated in Python. The dive initiation point is set at longitude 16°30'0''E and latitude 37°30'0''N. The direction of ocean current is northwest to southeast, and the size of ocean current decreases with increasing depth, and the seabed topography is relatively gentle on a small scale [5–8].

Through repeated simulations, the following results (**Figure 1** and **Figure 2**) are obtained:

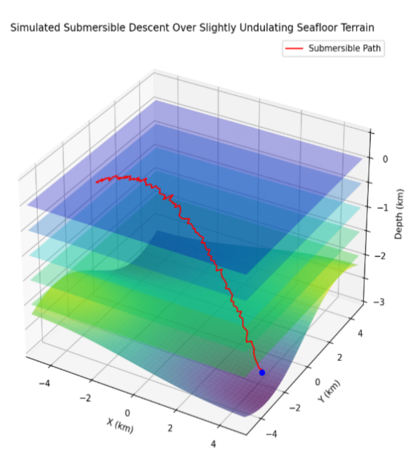


Figure 1. Submersible diving paths

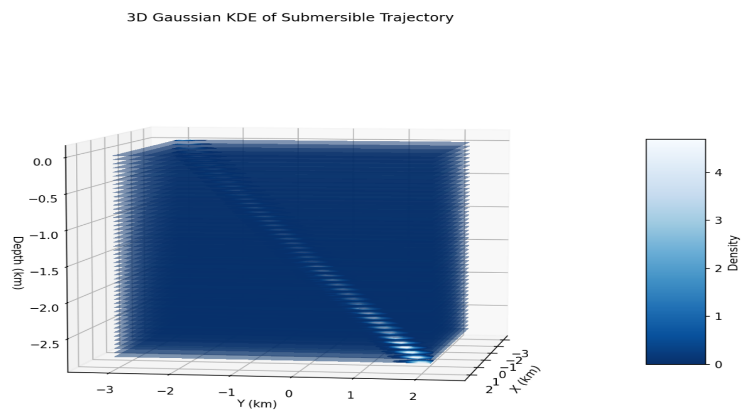


Figure 2. Probability density of submersible position

Based on these results, the final approximate position of the submersible can be estimated. As illustrated, the trajectory is significantly influenced by ocean currents. The predicted final location is confined to a small, roughly elliptical area with a high probability density.

Positional uncertainty arises from environmental variability and the submersible's own propulsion. Deviations in salinity, temperature, and current velocity from dataset averages contribute to this uncertainty. Consequently, the submersible requires a sensor suite including salinity, temperature, pressure, and flow sensors for real-time environmental monitoring. To account for propulsion-induced uncertainty, real-time monitoring of engine status and thrust direction is essential.

4. Model II: Two-dimensional kernel density estimation search model

4.1. Two-dimensional kernel density estimation of submersible

The process of searching for a submersible is related to the location of the submersible. With the distribution of the final position of the submersible obtained from Model I, two-dimensional kernel density estimation (2D KDE) is used, and the scatter plot of the point is converted into a usable probability density [9–10].

Given a set of samples $X=\{x_1,x_2,\ldots,x_n\}$ that are all two-dimensional vectors and taken from the same continuous distribution $f(x)$, the kernel density estimated at any point x is:

$$\widehat{f_h(\boldsymbol{x})} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h^2}K\left(\frac{\boldsymbol{x}-\boldsymbol{X}_i}{h}\right) \tag{4.1}$$

Where h is bandwidth, which uses Scott's Rule. K is a kernel function defined in two-dimensional space. $K{:}R^2 \to R_+$, satisfying:

$$K(\boldsymbol{x}) \geq 0, \int K(\boldsymbol{x})d\boldsymbol{u} = 1 \tag{4.2}$$

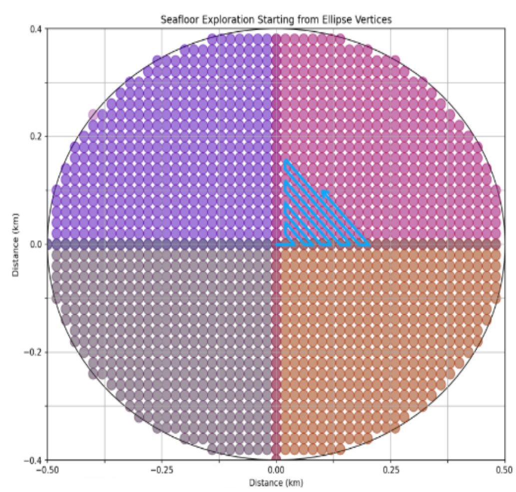
In this question, the coordinates of the submersible are represented by $p_r(x,y)$ i.e. p, so the formula is:

$$\widehat{f_h(\boldsymbol{p})} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h^2}K\left(\frac{\boldsymbol{x}-\boldsymbol{P}_i}{h}\right) \tag{4.3}$$

The resulting probability boundary is observed to approximate an elliptical shape. After normalization and simplification, the final search area is reduced to an ellipse. It is worth noting that the probability of submarines appearing is higher as they approach the center of the ellipse.

4.2. Submersible search model

According to the above results, the probability of finding the submersible decreases as the distance from the center of the ellipse increases. This elliptical region is partitioned into four sectors along its axes, with each sector assigned to an autonomous search robot initiating exploration from the center outward. This search method can search the most likely location of the submersible in the shortest time. This method ensures the separation of coverage areas between robots while covering the entire elliptical area as much as possible. The searching process is shown in the following **Figure 3**.



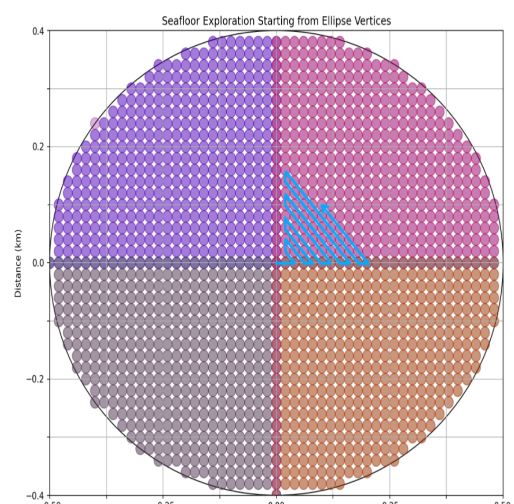


Figure 3. Searching process

Assuming a robot has a moving speed of $v_r(m/s)$ and a detectable radius of $r(m)$. The time t_i of the movement to the next location is

$$t_i = \frac{\sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}}{v_r} \tag{4.4}$$

Where x_i and y_i is the location of a circle P_{ci}. Total searching time T_s is

$$T_s = \sum_{i=1}^{n-1} t_i \tag{4.5}$$

Where n is the circle number in a quarter of the elliptical.

The probability of finding the submersible p_s is the ratio of the product of the area of the search area at T_s and the probability density of the probability of the entire ellipse 1.

$$p_s = \frac{4\sum_{i=1}^{k} \pi r^2 \times p_i}{1} \tag{4.6}$$

Where k is the number of points detected within a quarter ellipse at time T_s. p_i is the probability at the i-th detection point.

4.3. Result of Model II

In order to accelerate the search speed for the crashed submersible and obtain an estimate of the probability of successful search, the Two-dimensional kernel density estimation model is combined with the search model.

After the calculation, when the robot speed and the search radius , the time to complete the search is 1.28 hours. The Curve of the probability of search success over time (**Figure 4**) is shown below. It can be found that when the time is 1.3 hours, the probability of success of the search is close to 100%.

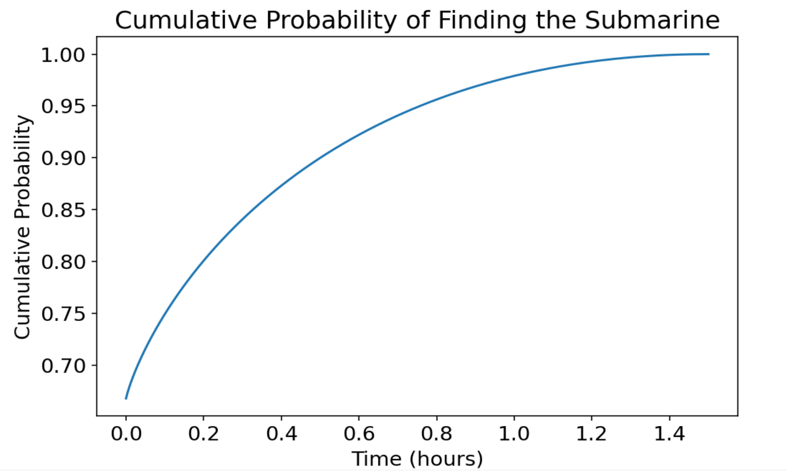


Figure 4. Curve of the probability of search success over time

5. Model III: Submersible position prediction model with multiple submersibles

5.1. Establish the model

Usually, in order to maximize profits, there is more than one submersible located in the same area. Studying the relationship between multiple submersibles is important for improving safety and rescue success rates. Due to the inconvenience of communication between the submersible and the host ship in the deep sea, higher power signal receiving and transmitting equipment is required to ensure communication quality. In order to save costs, submersible located in multiple submersible centers was chosen as the communication hub. communicates with - simultaneously, measures precise positions with each other, summarizes them, and reports them to the host ship. At the same time, the submersible -also try to synchronize its own data to the host ship. Therefore, the submersible network is established, and host ship only needs to use sonar to determine the position of rather than all of the submersibles, which makes it more accurate.

When one of the submersibles loses contact, its location can be accurately measured. Assuming a submersible (denoted as *Sn*) loses communication and requires rescue operations, modifications are implemented to Model I based on its last known precise position .

5.2. Result of Model III

It is assumed that the submersible lost contact at a depth of 1500 meters in the same sea region in Model I, and used the precise position before losing contact as a starting point to obtain multiple simulation results. The result is shown in **Figure 5** below.

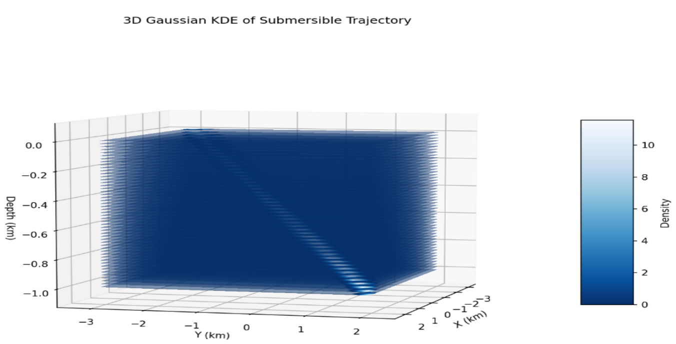


Figure 5. Result of Model III

As can be seen from the figure, the predicted area of the final possible location of the submersible is significantly smaller than that of **Figure 2**, which proves the accuracy of the model.

6. Conclusion

The model uses the latest, accurate Marine data to ensure the reliability of the results. After using the most important parameters, it can predict the position of the submersible more accurately and has a higher use value. In order to minimize search time, the optimal search path is selected according to the probability density of the submersible distribution, so the most likely location of the submersible can be searched in the shortest time. With the assistance of other submersibles, the position of the lost submersibles can be more precise. However, the uncertainty associated with the positions of other submersibles was not considered in this analysis. The results of the model are the most optimistic scenario, and there will be deviations from the facts.

Disclosure statement

The authors declare no conflict of interest.

References

[1] Li DJ, Zhao QS, He CR, et al., 2021, Motion Simulation Calculation and Test Verification of Deep Sea Manned Submersible. Ship Science and Technology, 43(6): 62–65.

[2] Zhu ZY, He MT, Wang YG, 2024, Prediction and Search for Lost Submarines Based on Monte Carlo Simulation. The 35th China Process Control Conference.

[3] Gill AE, 1982, Appendix Three — Properties of Seawater. International Geophysics (Vol. 30, pp. 599-603). Academic Press, Cambridge. https://doi.org/10.1016/S0074-6142(08)60041-8

[4] Wieselsberger C, 1922, New Data on the Laws of Fluid Resistance. National Advisory Committee for Aeronautics.

[5] NetCDF Climate and Forecast (CF) Metadata Convention, 2023, The GEBCO_2023 NetCDF Files are Provided in NetCDF 4 Format and Conform to the NetCDF Climate and Forecast (CF) Metadata Convention v1.6 (http://cfconventions.org/).

[6] Cheng LK, Trenberth J, Fasullo T, et al., 2017, Improved Estimates of Ocean Heat Content from 1960 to 2015. Science Advances, 2017(3): e1601545. https://advances.sciencemag.org/content/3/3/e1601545

[7] Li G, Cheng L, Pan Y, et al., 2023, A Global Gridded Ocean Salinity Dataset with 0.5° Horizontal Resolution since 1960 for the Upper 2000 m. Frontiers in Marine Science, 2023(10): 1–20. https://doi.org/10.3389/fmars.2023.1108919

[8] Tian T, Cheng L, Wang G, et al., 2022, Reconstructing Ocean Subsurface Salinity at High Resolution using a Machine Learning Approach. Earth System Science Data, 2022(14): 5037–5060. https://doi.org/10.5194/essd-14-5037-2022

[9] Silverman BW, 1986, Density Estimation for Statistics and Data Analysis. Chapman and Hall, London.

[10] Bashtannyk DM, Hyndman RJ, 2001, Bandwidth Selection for Kernel Conditional Density Estimation. Computational Statistics & Data Analysis, 2001(36): 279–298.

Publisher's note

Bio-Byword Scientific Publishing remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.